Extensions to the Skipgram Model

VL Embeddings

Uni Heidelberg

SS 2019

The SkipGram model

- Objective: Find word representations that are useful for predicting the surrounding words in a sentence or a document
- More formally:

$$-\frac{1}{T}\sum_{t=1}^{T}\sum_{-m\leq j\leq m, j\neq 0} log\ \ p(w_{t+1}|w_t) \tag{1}$$

where $$p(w_o|w_c) = \frac{exp(v_{w_o}^\top v_{w_c})}{\sum_{j=1}^{V} exp(v_j^\top v_{w_c})}$$ Softmax

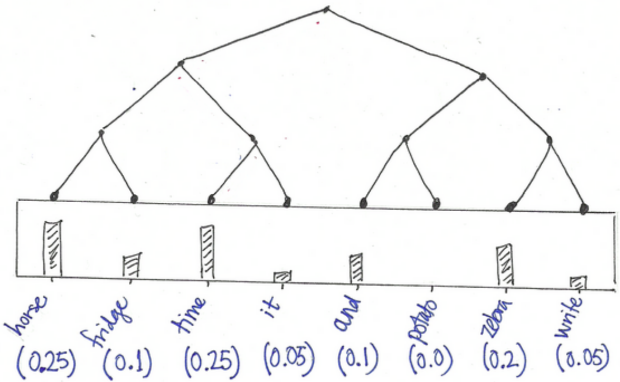
- All parameters need to be updated at every step
- Impractical: cost of computing $p(w_o|w_c)$ is proportional to V

Hierarchical Softmax

Computationally efficient approximation of the full softmax

- First introduced by Morin and Bengio (2005)
- Instead of evaluating V output nodes, we evaluate only $log_2(V)$ nodes
- How does it work?
  - binary tree representation of output layer where all words in vocab V are leaf nodes
  - for each node, represent the relative probabilities of its child nodes
  - random walk that assigns probabilities to words

Hierarchical Softmax



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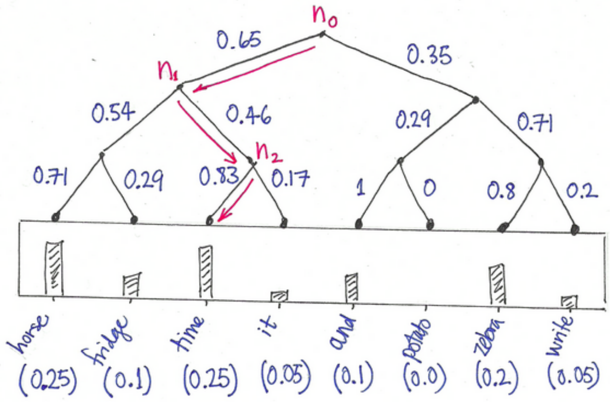
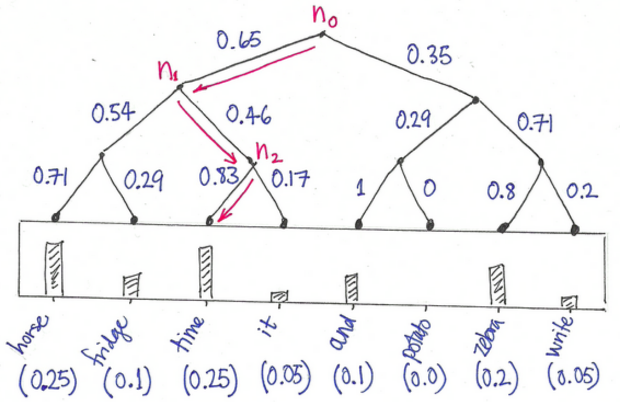
Hierarchical Softmax



For each node, represent the relative probabilities of its child nodes: transition probabilities to the children are given by the proportions of total probability mass in the subtree of its left- vs its right child

Hierarchical Softmax



Relative probabilities define a random walk that assigns probabilities to leaf nodes (words)

Hierarchical Softmax

- Probability for each word is result of a sequence of binary decisions
- For example

$$p(time|C) = P_{n_0}(left|C)P_{n_1}(right|C)P_{n_2}(left|C)$$

where $P_n(right|C)$ is the probability of choosing the right child when transitioning from node n

- There are only 2 outcomes, therefore

$$P_n(right|C) = 1 - P_n(left|C)$$

Hierarchical Softmax

But where does the tree come from?

- Different approaches in the literature:
  - Morin and Bengio (2005)
    - binary tree based on the IS-A relation in WordNet
  - Mnih and Hinton (2009)
    - boot-strapping method: hierarchical language model with a simple feature-based algorithm for automatic construction of word trees from data
  - Mikolov et al. (2013)
    - Huffman tree

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  - **Mikolov et al. (2013)**
    - **Huffman tree**

Hierarchical Softmax

Huffman trees (Mikolov et al. 2013)

- often used for loss-less data compression (Huffman 1952)
  - minimise expected path length from root to leaf

$\Rightarrow$ thereby minimising the expected number of parameter updates

Image from http://building-babylon.net/2017/08/01/hierarchical-softmax/

Hierarchical Softmax

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word	count
fat	3
fridge	2
zebra	1
potato	3
and	14
in	7
today	4
kangaroo	2

$\longrightarrow$

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Hierarchical softmax reduces number of parameters from V to $log_2(V)$

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Hierarchical Softmax

- Each word w can be reached by a path from the root node
- Average $L(w)$ is $log(V)$
- Assigns short codes to frequent words $\rightarrow$ fast training

Old

$$p(w_o|w_c) = \frac{exp(v_{w_o}^{\top} v_{w_c})}{\sum_{j=1}^{V} exp(v_j^{\top} v_{w_c})} \quad (2)$$

New

$$p(w|w_c) = \prod_{j=1}^{L(w)-1} \sigma({v'_{n(w,j)}}^{\top} v_{w_c}) \quad (3)$$

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- two representations (v_{w_c}, v_{w_o}) for each word w

New

$$p(w|w_c) = \prod_{j=1}^{L(w)-1} \sigma({v'_{n(w,j)}}^\top v_{w_c}) \tag{3}$$

- one representation for each word w and for each inner node v'_n

Hierarchical Softmax

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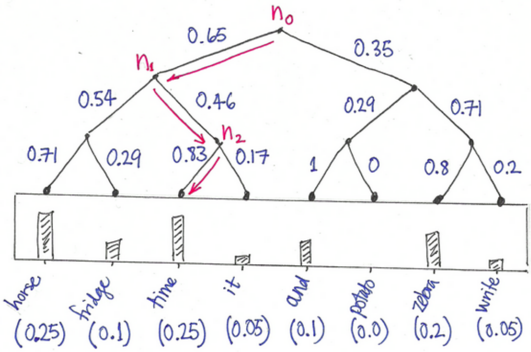

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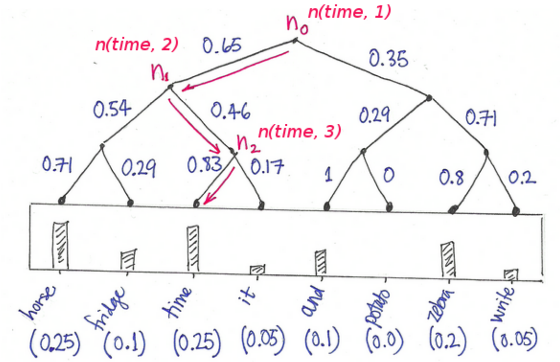

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$$\sum_{w=1}^{V} p(w|w_c) = 1 \tag{4}$$

$\Rightarrow$ implies that the cost of computing $log\ p(w_o|w_c)$ and $\nabla log\ p(w_o|w_c)$ is proportional to $L(w_o)$, which, on average, is $log(V)$

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Hierarchical Softmax – Sum-up

- Problem with Softmax:
  - cost of computing $p(w_o|w_c)$ is proportional to V
- Solution: Hierarchical Softmax
  - computationally efficient approximation of full Softmax
  - word2vec uses **Huffman trees** to implement Hierarchical Softmax
  - other tree representations are also possible (see Morin & Bengio 2005, Mnih & Hinton 2009)

Negative Sampling

Can we do better?

- Instead of summarising over all contexts in the corpus, create artificial negative samples

Goal: sample context words v_o that are unlikely to occur with v_c

- Generate the set of random (v_c, v_o) pairs, assuming they are all incorrect $\Rightarrow$ randomly sampled negative examples

Skip-Gram with Negative Sampling

- Given a pair (v_c, v_o) of word and context
  - $p(D = 1 | v_c, v_o)$ if $(v_c, v_o) \in D$

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$$argmax_\theta \sum_{(v_c, v_o) \in D} log\ p(D = 1|v_c, v_o; \theta)$$

Skip-Gram with Negative Sampling (II)

- We can define $p(D = 1|v_o, v_c; \theta)$:

$$p(D = 1|v_c, v_o; \theta) = \frac{1}{1+e^{-v_o \cdot v_c}}$$ sigmoid function

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- Training objective with negative sampling:

$$argmax_{v_c, v_o} \Bigg(\prod_{(v_c, v_o) \in D} p(D=1|v_o, v_c) \prod_{(v_c, v_o) \in D'} p(D=0|v_o, v_c) \Bigg) =$$

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$$argmax_{v_c, v_o} \Big(\sum_{(v_c, v_o) \in D} log\ \sigma(v_o \cdot v_c) + \sum_{(v_c, v_o) \in D'} log\ \sigma(-v_o \cdot v_c) \Big)$$

Skip-Gram with Negative Sampling (III)

- Online training using Stochastic Gradient Descent

$$J(\theta) = \frac{1}{T}\sum_{t=1}^{T} J_t(\theta)$$

$$J_t(\theta) = log\ \sigma(v_o^\top v_c) + \sum_{i=1}^{k} \mathbb{E}_{w_i \sim P_n(w)}[log\ \sigma(-v_{w_i}^\top v_c)]$$

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maximise probability of seen word pairs

minimise probability of unseen word pairs

Skip-Gram with Negative Sampling (IV)

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- For each $(v_c, v_o) \in D$ generate n samples $(v_c, v_{o_1}), \ldots, (v_c, v_{o_n})$ where
    - n is a hyperparameter
    - each v_{o_j} is drawn according to its unigram distribution raised to the 3/4 power $\quad P(w) = U(w)^{\frac{3}{4}}/Z$
      (causes less frequent words to be sampled more often)

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$\Rightarrow$ observed word pairs will have similar embeddings
$\Rightarrow$ unobserved word pairs will be scattered in space

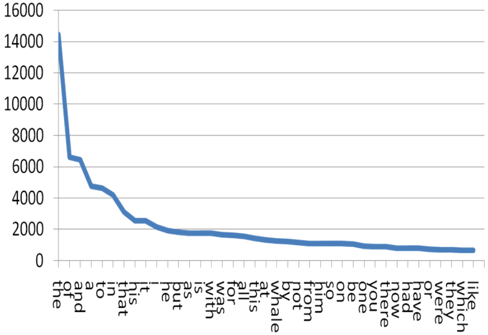
Skip-Gram with Negative Sampling (IV)

How many samples? Impact of sample size k

- 2 functions of k:
  1. better estimate of distribution of negative examples: higher k means more data and better estimation
  2. k acts as a prior on the probability of observing positive examples: higher $k \rightarrow$ negative examples more probable

Subsampling of frequent words

- In large corpora: Zipfian distribution
  - few words with very high frequency
  - many words with very low frequency



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  France, capital $\rightarrow$ more informative than *the, of*

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  France, capital $\rightarrow$ more informative than *the, of*
- Counter the imbalance between rare and frequent words

Subsampling of frequent words

- Simple subsampling approach:
  - Discard word w_i in the training set with probability

$$P(w_i) = 1 - \sqrt{\frac{t}{f(w_i)}} \tag{5}$$

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- Subsampling accelerates learning and significantly improves accuracy of embeddings for rare words

Sum-up: Extensions to the Skipgram model

Mikolov et al. (2013): Distributed Representations of Words and Phrases and their Compositionality

- More efficient training
- Higher quality word vectors

    - Training with negative sampling results in faster training and better vector representations for frequent words
    - Subsampling of frequent words improves training speed and accuracy for rare words
    - Extension from word-based to phrase vectors ($\rightarrow$ session on compositionality)

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